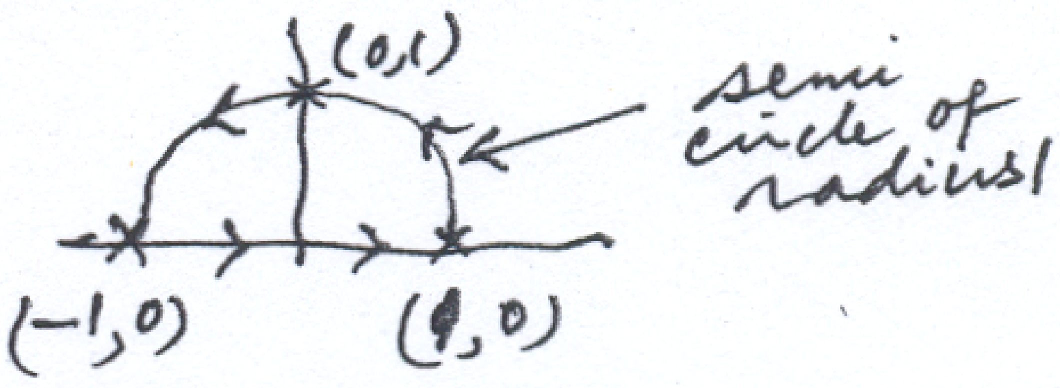
Analysis III / Mid-term exam / 2004-2005

Answer all questions. The maximum you can score is 60. You may use your classroom notes.

1) Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $F(x,y) = (x^2, y^2)$. Calculate $\int_\gamma \vec{F} \cdot d\vec{s}$, where γ is



(10 marks)

2) a) Locate the poles of the function f defined on $\mathbb{C}$ by $f(z) = \dfrac{z-3}{(z^2-9)^2(z+1)}$. What are the orders of these poles? If γ is the positively oriented circle of radius 20, centred at $(1,0)$, what is $\int_\gamma f(z)\,dz$?

(5 + 10)

b) Let $f(z) = \dfrac{1}{(z+1)(z+2)}$. If one writes $f(z)$ as a power series around i (i.e as powers of $(z-i)$), what is the radius of convergence of this power series? If $f(z) = \sum_{j=0}^{\infty} a_j (z-i)^j$, write down a_0, a_1 and a_2.

(5+10)

3) Calculate the Fourier series of $f(x) = |\sin x|$. Find an N s.t. $\int_{-\pi}^{\pi} |f(x) - S_N^f(x)|^2\,dx \leq \frac{1}{100}$

(10+10)

4) Let $\hat{f}(n) = e^{-5\log(n^4+1)}$. Decide if f is a C^∞ function or not. Justify your answer.

(5)

5) Let $f(x) = \sum_{n=1}^{\infty} n^2 e^{-n^2 x}$. Prove that the series on the RHS converges uniformly in $[\alpha, \infty)$ where $\alpha > 0$.

(5)